

Tutorial 2. 21 Sep 2015.

1. Review of some elementary functions.

a) $f(z) = \frac{i}{z}$. $D = \{z \mid \text{Im} z > 1\}$

b) $f(z) = iz + (3+4i)$. $D = \{z \mid |z| \leq 1\}$

c) $f(z) = \frac{z+2i}{3z+4}$. $D = \{z \in \mathbb{R}\}$ (Möbius Transformation).

2. Find a Möbius Transformation f s.t. f maps D_1 onto D_2 .

a) $D_1 = \{z \mid |z| \leq 1\}$. $D_2 = \{z \mid |z-1-i| \leq 2\}$

b) $D_1 = \{z \mid |z| \leq 1\}$. $D_2 = \{z \mid |z-1-i| \geq 2\}$

c) $D_1 = \{z \mid |z| \leq 1\}$. $D_2 = \{z \mid \text{Im} z \geq 0\}$

Hint: Implicit form.

a) $f(z) = \frac{z}{z+1+i}$

b) $f(z) = \frac{z}{z} + 1+i = \frac{z+(1+i)z}{z}$

c) $f(z) = \frac{z-1}{z+1} \frac{i+1}{i-1}$

3. Find ~~the~~ a real part for the following differentiable function $f(z)$

$= u+iv$ for $z = x+iy \in \mathbb{D}$

a) $v = e^{2x} \sin 2y$

b) $v = 2xy$

Hint: a) e^{2z}

b) z^2

Cauchy-Riemann Equation.